ON A CRITERION OF SUBSTRATUM HOMOGENEITY (OR HETEROGENEITY) IN FIELD EXPERIMENTS

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I. INTRODUCTORY REMARKS

EVERY ONE who has had practical experience in variety or fertilizer tests or in any other experiments involving the comparison of field plots must have been impressed by the great difficulty of securing tracts with uniform soil for their cultures.

A careful examination of the agricultural literature bearing on the question of variety tests will reveal many cases in which the experimenters have noted the difficulty of securing a uniform substratum, or in which there is internal evidence for the influence of substratum heterogeneity upon the result.

For example, in 1894–1895 tests of varieties of wheat were made on 77 plots at the University of Illinois.[1] As a check on the other strains, the variety known as Valley was sown on nine different plots "well distributed over the area sown."

> . . . the yields of this variety varied from 11.7 bushels to 24.1 bushels, an average of 19 bushels which is remarkably close to the average of all the varieties. It is again remarkable that but eight yields were above the highest of the Valley, and but three below the lowest of the same variety, . . .

The only reasonable explanations that can be given for such results are either (*a*) that the plots were so small that the results are due purely and simply to the errors of random sampling, or (*b*) that the wide divergences in the

[1] *Bull. Univ. Ill. Agr. Exp. Sta.*, 41, 1896.

results for the individual variety are due to substratum heterogeneity.

In either case, the results secured are obviously worthless as indicating differences in the value of the individual varieties.

Seventeen years ago, Larsen[2] reached the conclusion that the results of experimental tests were much more exact when a given area is divided into a large number of small plots upon which the tests are made than when it is divided into a few larger plots.

Hall[3] has laid great emphasis upon irregularities of experimental fields. Mercer and Hall in their interesting paper on "The Experimental Error of Field Trials"[4] discuss at considerable length various phases of the influence of soil heterogeneity upon field results. In an appendix to their paper, Student[5] takes up the problem of the method of arranging plots so as to utilize to the best advantage a given area of land in testing two varieties.[6]

The influence of substratum heterogeneity is also readily seen in Montgomery's interesting experimental data for wheat.[7]

Indeed, it is quite possible that without special precautions irregularities in the substratum may have greater influence upon the numerical results of an experiment than the factors which the investigator is seeking to compare. Elsewhere[8] I have shown that the differentiation

[2] Larsen, B. R., "Andra nordska Landbrakskongressen i Stockholm," 1897, I, p. 72; *fide* G. Holtermark and B. R. Larsen, *Lanwirtschaftl. Versuch-Stationen*, 65, 1, 1907.

[3] Hall, A. D., "The Experimental Error of Field Trials," *Journ. Board Agr. Great Britain*, 16, 365–370, 1909.

[4] *Journ. Agr. Sci.*, 4, 107–127, 1911.

[5] Student, *Journ. Agr. Sci.*, 4, 128–132, 1911.

[6] For several years, I have in careful tests labelled each seed individually and scattered them at random over the field to eliminate the influence of soil heterogeneity

[7] Montgomery, E. G., "Variation in Yield and Method of Arranging Plots to Secure Comparative Results," *Ann. Rep. Neb. Agr. Sta.*, 25, 164–180, 1912.

[8] Harris, J. Arthur, "An Illustration of the Influence of Substratum Heterogeneity upon Experimental Results," *Science*, N. S., 38, 345–346. 1913.

in an apparently uniform garden plot may be sufficient to mask entirely the influence of the weight of the seed (*Phaseolus vulgaris*) planted upon the size of the plant (as measured by the number of pods) produced. It is very probable that certain pure-line experiments and conclusions are entirely invalidated by the fact that the influence of irregularities in the substratum were not sufficiently guarded against.[9]

Several authors have tried to obtain some measure of, or some corrective term for, substratum heterogeneity. For example, Mercer and Hall (*loc. cit.*) have plotted the yields across the field in both directions. Such methods, however, give but a very imperfect idea of irregularities in the soil. Heterogeneity is perhaps more likely to occur as a spotting of the field than as a relatively uniform change from one side to the other. This is clearly indicated in the diagrams published by Montgomery. The mere plotting of yields in any line across the field can not adequately take into account such irregularities. Furthermore, some quantitative measure (and some probable error of this measure) of the amount of irregularity, not merely a demonstration of its existence, is required.

Some generally applicable measure of the degree of homogeneity of the soil of a field (as shown by actual capacity for crop production) seems highly desirable. Such a criterion should be universally applicable, comparable from species to species, character to character or experiment to experiment, and easy to calculate.

I believe we may proceed as follows. Suppose a field divided into N small plots all planted to the same variety of plants. Let p be the yield of an individual plot. The variability of p may be due purely and simply to chance, since the individuals of any variety are variable and the size of the plots is small, or it may be due in part to differentiation in the substratum. If the irregularities in the experimental field are so large as to influence the yield of

[9] See "The Distribution of Pure Line Means," AMER. NAT., 45, 686–700, 1911.

areas larger than single plots[10] they will tend to bring about a similarity of adjoining plots, some groups tending to yield higher than the average, others lower.

Now let the yields of these units be grouped into m larger plots, C_p, each of n contiguous ultimate units, p. The correlation between the p's of the same combination plot, C_p, will furnish a measure (on the scale of 0 to 1) of the differentiation of the substratum as expressed in capacity for crop production. If this correlation be sensibly 0, the irregularities of the field are not so great as to influence in the same direction the yields of neighboring small plots. As substratum heterogeneity becomes greater, the correlation will also increase. The size of the coefficient obtained will depend somewhat upon the nature of the characters measured, somewhat upon the species grown, and somewhat upon the size of the ultimate and combination plots. A knowledge of the values of the correlation to be expected must be determined empirically.

Fortunately, very simple formulæ are now available for calculating such coefficients.[11]

Let S indicate a summation for all the ultimate or combination plots of the field under consideration, as may be indicated by the capital C_p or lower case p. Then in our present notation which is as much simplified as possible for the special purposes of this discussion

$$r_{p_1p_2} = \frac{\{[S(C_p^2) - S(p^2)]/m[n(n-1)]\} - \bar{p}^2}{\sigma_p^2}$$

where $\bar{P}$ is the average yield of the ultimate plots and σ_p their variability, and n is constant throughout the m combination plots.[12]

10 Irregularities of soil influencing the plants of only a single small plot may in most work be left out of account, since they are of the kind to which differences between individual plants are to a considerable extent due, and are common to all the plots of a field.

11 Harris, J. Arthur, "On the Calculation of Intra-class and Inter-class Coefficients of Correlation from Class Moments when the Number of Possible Combinations is Large," *Biometrika*, 9, 446–472, 1913.

12 For the benefit of those who are frightened by formulæ, it may be paraphrased as follows: One merely adds together the yields of a chosen

Ultimately, I hope to present experimental data of my own bearing on this problem. For the present, the method is admirably illustrated by a number of published records.

II. Illustrations of Method

A. Cases in which the Combination Plots are Equal in Size

Illustration 1. *Influence of substratum heterogeneity on yield of experimental plots of mangolds.*

TABLE I

Yield of Combination Plots for Mangolds, Obtained by Combining the Entries of Map A by Fours as Indicated by the Heavier Lines

1,209 172	1,175 183	1,215 171	1,239 175	1,276 205
1,250 185	1,321 191	1,274 187	1,293 184	1,310 207
1,204 159	1,333 188	1,268 172	1,290 185	1,268 200
1,300 172	1,272 177	1,222 167	1,272 173	1,388 215
1,385 193	1,375 194	1,314 193	1,260 180	1,373 219
1,380 204	1,387 202	1,309 177	1,314 188	1,380 229
1,320 180	1,295 188	1,304 187	1,332 194	1,397 226
1,331 183	1,264 183	1,310 188	1,325 183	1,337 203
1,404 194	1,325 190	1,334 190	1,335 192	1,312 211
1,418 193	1,373 196	1,339 189	1,403 198	1,401 226

number of contiguous p plots to form a number m of C_p plots. The sum of the squares of p is subtracted from the sum of the squares of C_p and the result divided by $m[n(n-1)]$ where n is the number of ultimate plots in each of the m combination plots. The quotient is reduced by subtracting the square of the mean yields of the ultimate plots, $\bar{p}$, and the remainder divided by the square of the standard deviation of yields of ultimate plots, σ_p^2. The quotient is the correlation between the yields of the ultimate units, p, of the same combination plot, C_p,—the measure of heterogeneity required. Thus the calculation of the criterion is very simple indeed.

NW	1	2	3	4	5	6	7	8	9	10
1	310 42	302 46	288 41	325 48	321 45	291 41	306 41	306 45	306 48	330 57
2	290 41	307 43	267 40	295 54	308 45	295 40	317 44	310 45	316 48	324 52
3	322 49	309 45	322 43	324 49	330 52	286 40	300 46	325 47	302 47	298 49
4	309 44	310 47	324 46	351 53	342 51	316 44	324 44	344 47	341 51	369 60
5	278 38	320 42	335 46	350 51	342 49	309 40	310 45	322 48	329 50	300 52
6	302 37	304 42	310 45	338 46	316 43	301 40	328 44	330 48	325 46	314 52
7	306 41	318 42	302 39	332 50	299 43	277 37	331 44	322 46	384 61	339 52
8	333 45	343 44	318 44	320 44	335 47	311 40	296 38	323 45	327 49	338 53
9	339 44	336 46	324 46	347 47	331 51	313 42	312 48	300 41	346 52	343 57
10	360 51	350 52	354 51	350 50	348 54	322 46	325 43	323 48	335 50	349 60
11	346 51	362 55	372 51	349 54	343 46	308 40	297 43	328 49	352 56	309 55
12	327 47	345 51	331 48	335 49	342 49	316 42	339 47	350 49	365 54	354 64
13	310 45	364 50	300 46	337 45	321 46	329 46	341 51	349 49	363 58	347 59
14	317 41	329 44	321 45	337 52	340 50	314 45	321 48	321 46	346 52	341 57
15	323 44	326 44	290 41	328 48	348 49	325 45	358 48	332 44	349 51	335 56
16	353 49	329 46	311 47	335 47	331 46	306 48	318 47	317 44	332 46	321 50
17	357 52	348 46	301 44	335 47	340 51	336 45	327 46	330 50	343 54	317 54
18	362 46	337 50	339 47	350 52	328 47	330 47	343 49	335 47	326 47	326 56
19	349 46	365 52	359 48	339 47	340 48	332 49	356 50	336 48	338 51	316 56
20	352 49	352 46	340 50	335 51	332 44	335 48	356 50	355 50	371 52	376 67

MAP A. Pounds per Plot of Roots and Leaves of Mangolds. Data of Mercer and Hall.

Map A represents the Rothamsted field of mangolds grown by Mercer and Hall (*loc. cit.*). The upper entries are for pounds of roots, the lower for pounds of leaves.

I now reduce the 200 areas to 50 by combining the adjoining plots by fours, as indicated by the heavier lines on the map. Thus for leaves the Southwest combination plot, C_p, is $67+52+56+51=226$. Table I gives the result.

This gives for roots:

$$S(p)=65715, \quad S(p^2)=21674871, \quad N=200,$$
$$\bar{p}=328.575, \quad \sigma_p{}^2=412.824,\text{[13]}$$
$$S(C_p{}^2)=86537439, \quad m[n(n-1)]=50\times4\times3=600,$$
$$[S(C_p{}^2)-S(p^2)]/m[n(n-1)]=108104.280,$$

and

$$r_{p_1p_2}=\frac{108104.280-(328.575)^2}{412.824}=.346\pm.042.\text{[14]}$$

The results for yield of leaves are

$$S(p)=9541, \quad S(p^2)=45941, \quad N=200,$$
$$\bar{p}=47.705, \quad \sigma_p{}^2=23.938,$$
$$S(C_p{}^2)=1832095, \quad m[n(n-1)]=50\times4\times3=600,$$
$$[S(C_p{}^2)-S(p^2)]/m[n(n-1)]=2286.923,$$

whence

$$r_{p_1p_2}=\frac{2286.923-(47.705)^2}{23.938}=.466\pm.037.$$

Illustration 2. *Influence of Substratum Heterogeneity upon the Yield of Straw and Grain in Experimental Plots of Wheat.*

[13] The standard deviation is most conveniently calculated in cases like the present, in which one requires the summed squares of actual values for other purposes from

$$\sigma_p{}^2=\Sigma(p^2)/N-[\Sigma(p)/N]^2.$$

[14] The probable errors have in all cases been calculated upon the actual, not the weighted, number of ultimate plots as N.

The wheat field of Mercer and Hall is divided into $25 \times 20 = 500$ plots, Map B. Combining the plots by fives from east to west and by fours from north to south, I have condensed this into $5 \times 5 = 25$ C_p plots, each of 20 ultimate plots as shown in Table II.

TABLE II

YIELDS OF COMBINATION PLOTS OF ROTHAMSTED WHEAT, 4×5 GROUPING. ORIGINAL AREAS SEPARATED BY DOUBLE LINES IN MAP B

82.89 139.36	83.05 132.41	78.63 122.84	78.76 120.53	74.70 114.58
78.15 130.60	84.34 140.31	75.61 120.11	80.32 119.27	74.87 112.21
79.80 133.31	84.70 149.58	74.94 125.27	81.50 133.28	77.34 120.09
84.36 142.79	82.42 147.74	73.60 131.80	71.35 121.18	75.81 122.02
85.19 147.95	84.56 146.78	82.25 138.42	68.52 120.09	76.69 124.88

Summing the actual yields and the squares of yields for the ultimate plots and the squares for the combination plots, I find the following values:

For wheat grain

$$S(p) = 1974.32, \quad S(p^2) = 7900.6790, \quad N = 500,$$

$$\bar{p} = 3.949, \quad \sigma_p{}^2 = .209600,$$

$$S(C_p{}^2) = 156419.3106, \quad m[n(n-1)] = 25 \times 20 \times 19 = 9{,}500,$$

$$[S(C_p{}^2) - S(p^2)]/m[n(n-1)] = 15.633540,$$

which leads to

$$r_{p_1p_2} = \frac{15.633540 - (3.949)^2}{.209600} = .186 \pm .029.$$

For wheat straw

$$S(p) = 3257.40, \quad S(p^2) = 21623.9802, \quad N = 500,$$

$$\bar{p} = 6.515, \quad \sigma_p{}^2 = .805341,$$

$$S(C_p{}^2 = 427479.9920, \quad m[n(n-1)] = 9500,$$

$$[S(C_p{}^2) - S(p^2)]/m[n(n-1)] = 42.721685,$$

NW	1	2	3	4	5	6	7	8	9	10	11	12
1	3.63 6.37	4.15 6.85	4.06 7.19	5.13 7.99	3.04 4.71	4.48 6.08	4.75 7.31	4.04 6.08	4.14 6.98	4.00 5.87	4.37 6.75	4.02 6.10
2	4.07 6.24	4.21 7.29	4.15 7.41	4.64 7.80	4.03 6.34	3.74 6.63	4.56 7.88	4.27 6.35	4.03 6.91	4.50 6.50	3.97 6.09	4.19 6.43
3	4.51 7.05	4.29 7.71	4.40 7.35	4.69 7.50	3.77 6.17	4.46 6.98	4.76 8.18	3.76 5.93	3.30 5.95	3.67 6.20	3.94 6.18	4.07 6.37
4	3.90 6.91	4.64 8.23	4.05 7.89	4.04 6.66	3.49 5.70	3.91 6.46	4.52 7.60	4.52 7.29	3.05 5.82	4.59 5.41	4.01 5.99	3.34 5.60
5	3.63 5.93	4.27 7.73	4.92 8.58	4.64 7.86	3.76 6.05	4.10 6.77	4.40 7.91	4.17 7.33	3.67 7.33	5.07 8.05	3.83 6.36	3.63 6.43
6	3.16 5.59	3.55 6.45	4.08 7.04	4.73 7.98	3.61 5.89	3.66 6.15	4.39 7.36	3.84 6.28	4.26 7.61	4.36 5.58	3.79 5.46	4.09 6.10
7	3.18 5.32	3.50 5.87	4.23 7.02	4.39 6.98	3.28 4.97	3.56 6.06	4.94 8.06	4.06 6.81	4.32 7.37	4.86 7.51	3.96 6.23	3.74 6.38
8	3.42 5.52	3.35 5.71	4.07 7.05	4.66 7.28	3.72 5.78	3.84 6.10	4.44 7.50	3.40 5.97	4.07 6.99	4.93 7.57	3.93 6.13	3.04 4.96
9	3.97 6.03	3.61 6.01	4.67 7.64	4.49 6.95	3.75 5.94	4.11 6.83	4.64 7.92	2.99 5.07	4.37 7.25	5.02 8.23	3.56 5.75	3.59 6.03
10	3.40 5.66	3.71 6.29	4.27 7.17	4.42 6.95	4.13 7.31	4.20 6.86	4.66 7.59	3.61 6.33	3.99 7.26	4.44 7.75	3.86 6.14	3.99 6.26
11	3.39 5.61	3.64 6.30	3.84 6.60	4.51 7.86	4.01 7.18	4.21 8.23	4.77 8.23	3.95 7.11	4.17 7.52	4.39 7.73	4.17 7.20	4.17 7.08
12	4.43 7.07	3.70 6.17	3.82 6.87	4.45 7.17	3.59 6.53	4.37 8.75	4.45 8.74	4.08 7.17	3.72 7.28	4.56 7.73	4.10 6.90	3.07 6.12
13	4.52 7.10	3.79 6.33	4.41 7.03	4.57 7.93	3.94 7.06	4.47 8.53	4.42 8.02	3.92 6.70	3.86 7.20	4.77 7.67	4.99 7.82	3.91 7.34
14	4.46 7.16	4.09 7.22	4.39 7.73	4.31 7.31	4.29 7.08	4.47 8.15	4.37 7.69	3.44 6.62	3.82 7.05	4.63 7.87	4.36 7.39	3.79 6.33
15	3.46 8.85	4.42 5.20	4.29 7.52	4.08 6.67	3.96 6.54	3.96 7.10	3.89 6.86	4.11 7.58	3.73 6.89	4.03 7.16	4.09 7.03	3.82 7.30
16	5.13 8.37	3.89 7.05	4.26 6.99	4.32 6.93	3.78 6.72	3.54 6.46	4.27 7.79	4.12 7.32	4.13 7.24	4.47 7.84	3.41 5.96	3.55 6.70
17	4.23 6.89	3.87 6.82	4.23 7.14	4.58 7.73	3.19 6.06	3.49 6.63	3.91 7.34	4.41 7.53	4.21 7.41	4.61 7.51	4.27 7.17	4.06 7.00
18	4.38 6.72	4.12 7.38	4.39 7.55	3.92 6.70	4.84 8.85	3.94 6.75	4.38 7.43	4.24 7.32	3.96 7.04	4.29 6.96	4.52 7.73	4.19 7.30
19	3.85 6.59	4.28 7.03	4.69 8.06	5.16 8.78	4.46 7.54	4.41 8.15	4.68 7.51	4.37 7.19	4.15 7.47	4.91 7.96	4.68 8.07	5.13 8.31
20	3.61 6.20	4.22 7.65	4.42 8.45	5.09 8.72	3.66 7.09	4.22 7.72	4.06 7.06	3.97 7.53	3.89 7.36	4.46 6.91	4.44 6.87	4.52 8.17

MAP B. Wheat Yields, Upper Figures Grains, Lower Figures Straw,

13	14	15	16	17	18	19	20	21	22	23	24	25
4.58 7.23	3.92 6.33	3.64 5.11	3.66 5.96	3.57 5.12	3.51 5.05	4.27 6.54	3.72 5.47	3.36 4.76	3.17 4.95	2.97 4.53	4.23 6.08	4.53 6.78
4.05 6.57	3.97 6.03	3.61 5.58	3.82 5.80	3.44 5.00	3.92 5.83	4.26 8.61	4.36 6.14	3.69 5.56	3.53 5.09	3.14 5.11	4.09 5.91	3.94 5.68
3.73 6.02	4.58 7.23	3.64 5.86	4.07 6.74	3.44 5.56	3.53 4.91	4.20 6.55	4.31 6.44	4.33 6.17	3.66 6.15	3.59 5.41	3.97 6.28	4.38 7.49
4.06 6.19	3.19 6.56	3.75 4.62	4.54 7.08	3.97 6.03	3.77 5.79	4.30 5.95	4.10 5.96	3.81 6.13	3.89 5.92	3.32 4.62	3.46 5.41	3.64 6.55
3.74 6.13	4.14 5.98	3.70 7.67	3.92 6.14	3.79 5.33	4.29 5.58	4.22 6.15	3.74 5.76	3.55 5.89	3.67 5.45	3.57 5.24	3.96 5.60	4.31 6.56
3.72 6.03	3.76 5.49	3.37 5.00	4.01 5.99	3.87 5.57	4.35 6.09	4.24 5.88	3.58 5.61	4.20 5.92	3.94 5.87	4.24 5.82	3.75 5.50	4.29 6.15
4.33 6.79	3.77 5.48	3.71 5.66	4.59 7.28	3.97 6.03	4.38 6.24	3.81 5.69	4.06 6.25	3.42 5.45	3.05 4.57	3.44 4.56	2.78 4.28	3.44 5.68
3.72 5.97	3.93 6.07	3.71 5.79	4.76 6.49	3.83 6.29	3.71 5.91	3.54 5.21	3.66 5.78	3.95 5.92	3.84 5.66	3.76 5.24	3.47 5.59	4.24 7.26
4.05 6.82	3.96 6.35	3.75 5.12	4.73 8.64	4.24 6.45	4.21 6.29	3.85 6.15	4.41 6.15	4.21 6.04	3.63 5.81	4.17 5.58	3.44 4.81	4.55 6.32
3.37 6.25	3.47 5.78	3.09 5.47	4.20 6.49	4.09 6.16	4.07 6.18	4.09 5.47	3.95 6.11	4.08 7.00	4.03 5.72	3.97 5.65	2.84 4.10	3.91 5.96
4.09 7.28	3.29 5.71	3.37 6.44	3.74 8.63	3.41 5.78	3.86 6.14	4.36 7.39	4.54 7.46	4.24 7.20	4.08 6.54	3.89 5.98	3.47 5.84	3.29 5.65
3.99 7.13	3.14 5.05	4.86 6.39	4.36 7.26	3.51 6.11	3.47 5.90	3.94 6.68	4.47 7.84	4.11 6.95	3.97 6.47	4.07 5.80	3.56 6.38	3.83 6.29
4.09 7.72	3.05 5.70	3.39 5.86	3.60 6.27	4.13 6.87	3.89 6.23	3.67 6.20	4.54 7.33	4.11 6.64	4.58 6.79	4.02 6.35	3.93 5.69	4.33 7.11
3.56 6.69	3.29 5.71	3.64 6.36	3.60 5.84	3.19 5.87	3.80 6.14	3.72 6.34	3.91 6.96	3.35 6.27	4.11 6.64	4.39 6.11	3.47 5.78	3.93 6.07
3.57 6.55	3.43 5.38	3.73 8.58	3.39 6.42	3.08 5.42	3.48 5.52	3.05 5.20	3.65 6.60	3.71 6.29	3.25 6.37	3.69 5.18	3.43 5.82	3.38 5.68
3.16 5.84	3.47 5.84	3.30 5.70	3.39 5.80	2.92 4.95	3.23 5.33	3.25 5.25	3.86 6.64	3.22 5.40	3.69 5.93	3.80 5.70	3.79 6.21	3.63 5.99
3.75 6.31	3.91 6.21	3.57 5.99	3.45 6.05	3.05 7.64	3.68 5.82	3.52 5.85	3.91 6.71	3.87 6.13	3.87 7.50	4.21 5.48	3.68 6.01	4.06 6.88
4.49 7.57	3.82 6.37	3.60 6.34	3.14 5.48	2.73 4.77	3.09 5.41	3.66 5.84	3.77 6.98	3.48 6.14	3.76 6.11	3.69 5.43	3.84 6.35	3.67 6.33
4.19 6.93	4.41 6.78	3.54 5.58	3.01 5.68	2.85 4.96	3.36 6.14	3.85 6.15	4.15 6.85	3.93 6.57	3.91 6.09	4.33 6.04	4.21 6.98	4.19 6.93
3.70 6.80	4.28 6.97	3.24 5.95	3.29 5.58	3.48 5.52	3.49 5.82	3.68 6.76	3.36 6.08	3.71 6.35	3.54 6.21	3.59 4.66	3.76 6.36	3.36 6.33

on Rothamsted Acre.

whence

$$r_{p_1p_2}'' = \frac{42.721685 - (6.515)^2}{.805341} = .343 \pm .027.$$

Illustration 3. *Influence of Substratum Heterogeneity upon Yield of Grain and Nitrogen Content in Experimental Plots of Wheat.*

Table III is condensed from Map C of Montgomery[15]

532 2.13	514 2.06	525 2.20	501 196	534 2 10	486 2.03	483 2.19	451 2.06	395 2.08	440 2.13	432 2.03	441 2.11	410′ 2.15	470 2.14
612 2.16	510 2.05	460 2.06	530 2.00	538 2.09	471 1.98	439 2.17	446 2.14	424 2 13	430 1.96	424 1.97	405 2.01	436 1.98	472 2.04
515 2.10	480 2.20	485 2.08	534 2.06	423 2.05	421 2.11	446 2.15	393 2.02	414 2.09	421 1.97	422 1.95	423 2.03	380 1.96	432 2.02
553 2.03	542 2.00	583 2.11	524 2.13	420 2.08	436 2.05	428 2.18	400 2.17	383 2.08	434 1.98	395 1.94	421 2.06	490 1.95	467 2.00
575 2.12	532 2.06	550 2.09	548 2.05	480 2.04	432 2.00	420 2.04	452 2.01	460 1.97	449 1.92	474 2.11	443 2.02	484 1.94	494 2.10
576 2.14	559 2.13	353 2.07	530 2.21	576 2.05	583 2.07	566 2.15	517 2.04	530 2.13	346 1.98	495 1.97	474 1.88	434 1.86	573 2.05
548 2.06	513 2.09	533 2.05	517 2.14	503 2.04	580 2.04	514 2.03	632 2.25	740 1.86	641 1.97	506 2.04	495 2.02	560 1.99	575 1.95
550 2.01	463 2.18	550 2.08	540 2.11	497 2.00	424 2.18	519 1.95	606 2.14	756 2.44	656 2.00	584 2.10	623 1.93	599 2.00	695 1.94
465 2.09	456 2.17	487 2.01	343 2.26	628 2.24	728 2.13	616 2.05	620 2.16	724 2.13	675 1.95	647 2.09	710 1.96	711 1.98	633 2.01
545 2.00	577 1.97	583 1.99	515 2.06	515 2.27	535 2.23	467 2.21	577 2.26	581 2.15	648 2.21	707 2.22	738 2.11	717 2.11	621 2.28
504 2.02	657 2.31	760 2.14	551 1.93	528 2.07	558 2.18	575 2.11	531 2.05	686 2.10	656 1.97	716 2.19	739 2.02	730 2.08	658 2.03
582 2.00	596 2.01	595 2.00	622 2.01	644 2.06	541 2.08	584 1.96	673 1.92	676 2.04	712 2.32	666 2.11	688 2.04	639 2.04	555 2.06
668 2.08	662 1.98	613 2.07	693 2.10	666 2.20	643 2.03	570 2.02	674 2.16	661 2.00	742 2.12	802 2.21	634 2.22	634 2.06	634 2.07
661 2.07	582 2.03	590 2.03	560 2.07	585 2.14	576 2.09	500 2.08	542 2.17	557 2.05	538 2.15	500 2.29	588 2.16	685 2.16	587 2.16
730 2.18	650 2.05	650 2.08	586 2.22	533 1.97	617 2.04	561 2.15	496 2.27	527 2.23	637 2.18	385 2.11	585 2.09	625 1.99	637 2.06
575 1.99	495 2.04	502 2.08	584 2.16	716 2.26	725 2.10	563 2.11	477 2.14	513 2.14	649 2.03	547 2.06	488 2.06	512 1.93	426 2.00

MAP C. Yields of Grain in Grams and Percentage of Nitrogen in Montgomery's Wheat Plots. The Upper Entries are Yield in Grams of Grain, the Lower are Percentage Nitrogen Content.

[15] Montgomery, E. G., "Variation in Yields and Method of Arranging Plots to Secure Comparative Results," *Ann. Rep. Neb. Agr. Exp. Sta.*, 25, 164–177, 1912.

by combining adjoining plots 2×2. The following are the numerical values.

For grains produced,

$$S(p) = 123429, \quad S(p^2) = 70112319, \quad N = 224,$$
$$\bar{p} = 551.022, \quad \sigma_p{}^2 = 9375.826,$$
$$S(C_p{}^2) = 277945243, \quad m[n(n-1)] = 642,$$
$$[S(C_p{}^2) - S(p^2)]/m[n(n-1)] = 309275.184,$$

whence

$$r_{p_1 p_2} = .603 \pm .029.$$

For percentage nitrogen,

$$S(p) = 465.29, \quad S(p^2) = 968.3721, \quad N = 224,$$
$$\bar{p} = 2.077187, \quad \sigma_p{}^2 = .008383,$$
$$S(C_p{}^2) = 3868.5047, \quad m[n(n-1)] = 672,$$
$$[S(C_p{}^2) - S(p^2)]/m[n(n-1)] = 4.315673,$$

and

$$r_{p_1 p_2} = .115 \pm .044.$$

TABLE III

COMBINATION PLOTS OF MONTGOMERY'S WHEAT, 2 × 2-FOLD GROUPING AS INDICATED BY HEAVY LINES IN MAP C

2,168 8.40	2,016 8.22	2,029 8.20	1,819 8.56	1,689 8.30	1,702 8.12	1,788 8.31
2,090 8.33	2,126 8.38	1,700 8.29	1,667 8.52	1,652 8.12	1,661 7.98	1,769 7.93
2,242 8.45	1,981 8.42	2,071 8.16	1,955 8.24	1,785 8.00	1,886 7.98	1,985 7.95
2,074 8.34	2,140 8.38	2,004 8.26	2,271 8.37	2,793 8.27	2,208 8.09	2,429 7.88
2,043 8.23	1,928 8.32	2,406 8.87	2,280 8.68	2,628 8.44	2,802 8.38	2,682 8.38
2,339 8.34	2,528 8.08	2,271 8.39	2,363 8.04	2,730 8.43	2,809 8.36	2,582 8.21
2,573 8.16	2,456 8.27	2,470 8.46	2,286 8.43	2,498 8.32	2,524 8.88	2,540 8.45
2,450 8.26	2,322 8.54	2,591 8.37	2,097 8.67	2,326 8.58	2,005 8.32	2,200 7.98

Illustration 4. *Influence of Substratum Heterogeneity upon the Yield of Experimental Plots of Timothy Hay.*

I take as a final illustration of the application of the criterion of substratum heterogeneity here proposed, the plot data for timothy hay published by Holtermarck and Larsen, *loc. cit.* By combining their plots into groups of 4 Table IV is secured,

$$S(p) = 4268.8, \quad S(p^2) = 77968.50, \quad N = 240,$$
$$\bar{p} = 17.787, \quad \sigma_p{}^2 = 8.503,$$
$$S(C_p{}^2) = 309491.48, \quad m[n(n-1)] = 720,$$

whence

$$r_{p_1p_2} = .609 \pm .027.$$

TABLE IV

COMBINATION PLOTS 2 × 2, SHOWING YIELDS OF TIMOTHY HAY SECURED IN THE EXPERIMENT OF LARSON

The original field is not mapped here

87.4	99.0	78.5	65.8	67.2	63.3
76.4	70.2	75.0	73.1	67.7	59.7
76.9	65.2	64.2	89.7	72.1	64.3
65.1	54.1	66.4	98.9	83.3	64.3
57.9	64.7	61.1	88.6	72.2	64.8
73.0	55.6	62.2	75.6	82.8	71.1
58.1	72.1	67.2	60.2	77.5	75.6
71.7	67.0	54.3	64.8	81.6	75.2
68.8	70.4	61.7	81.2	72.8	61.4
77.5	79.6	66.9	83.4	73.9	68.5

B. Cases in which the Combination Plots Vary in Size

In the foregoing illustration the combination plots have been of uniform size, *i. e.,* have contained each the same number of ultimate plots. It may be desirable or necessary to have some of the combination plots smaller than the others. Thus the wheat field of Mercer and Hall is

laid out in a 20×25 manner. This permits only 2×5, 4×5 or 5×5 combinations of the same size throughout. Montgomery's experiment comprises an area of 16×14 plots which may be combined in only 2×2 or 4×2 equal areas suitable for calculation. In each of these cases other groupings are desirable.

The formulæ are quite applicable to such cases: the arithmetical routine is merely a little longer. The formula is again

$$r_{p_1 p_2} = \frac{\{[S(C_p^2) - S(p^2)]/S[n(n-1)]\} - \bar{p}^2}{\sigma_p^2},$$

but $\bar{p}$ and σ_p are obtained by a $(n-1)$-fold weighting of the plots,[16] where n is the number of ultimate plots in the combination plot to which any p may be assigned, *i. e.*,

$$\bar{p} = S[(n-1)p]/S[n(n-1)],$$

$$\sigma_p^2 = \frac{S[(n-1)p^2]}{S[n(n-1)]} - \left(\frac{S[(n-1)p]}{S[n(n-1)]}\right)^2.$$

The point may be illustrated in detail on the wheat data of Mercer and Hall. I adopt a combination by twos from north to south, *i. e.*, arrange the data in 10 rows of combination plots instead of 20 rows of ultimate plots. From east to west there are 25 rows of ultimate plots; these can be only reduced to a 2×2-fold grouping for the first 22 rows. The lines of division are indicated on Map B by marginal arrows.

Row 23–25 must be thrown into combination plots each of 6 units. The possible permutations within a combination plot are $1/2\, n(n-1)$, but since the surfaces are rendered symmetrical, the total permutations for the whole field is $S[n(n-1)]$. There are only two sizes of combination plots, of which 110 have 4 and 10 have 6 ultimate plots each. Thus the weighted population N is

16 That is, each ultimate plot is multiplied by the number less one of the plots in the combination plot to which it is assigned.

$$S[n(n-1)] = (110 \times 4 \times 3) + (10 \times 6 \times 5) = 1620.$$

In the calculation of the weighted means and standard deviations each entry, and the square of each entry, in the first 22 rows must be weighted in an $(n-1)$-fold $=3$-fold manner, while those for the last three rows must be weighted in a 5-fold manner.[17]

The numerical values are:

For grain,

$$S[(n-1)p] = 6378.72, \quad S[(n-1)p^2] = 25452.4154,$$
$$\bar{p} = 3.937, \quad \sigma_p{}^2 = .207610,$$

$$S(C_p{}^2) = 33129.7080, \quad S(p^2) = 7900.6790,$$

whence

$$r_{p_1p_2} = .354 \pm .026.$$

Note that $S(p^2)$ is constant for all groupings.

For straw,

$$S[(n-1)p] = 10474.52, \quad S[(n-1)p^2] = 69042.7194,$$
$$\bar{p} = 6.466, \quad \sigma_p{}^2 = .813000,$$
$$S(C_p{}^2) = 89985.8976, \quad S(p^2) = 21623.9802,$$

whence

$$r_{p_1p_2} = .479 \pm .023.$$

Weighting has not materially changed the physical constants from the values given under illustration 2 above. The reasons for the conspicuous differences in the correlations will be taken up presently.

Montgomery's wheat data have been grouped into 2×2-fold combination plots in the illustration above. If we again combine the entries of Table III by twos, beginning at the upper left-hand corner, we have 12 combination plots each 4×4, or of 16 ultimate plots, and 4 combina-

17 Since each individual ultimate plot is compared once as a first (or as a second) number of a pair with every plot classed with it, the weighting of the individual plots for means and standard deviations is an $(n-1)$-fold one.

tion plots each of $2 \times 4 = 8$ ultimate plots. The method of dividing up the field is indicated by the marginal arrows on Map C.

$$S[n(n-1)] = (12 \times 16 \times 15) + (4 \times 8 \times 7) = 3104.$$

For grain,

$$S[(n-1)p] = 1707635, \quad S[(n-1)p^2] = 9683408.57$$
$$\bar{p} = 550.140, \quad \sigma_p{}^2 = 9311.307,$$
$$S(C_p{}^2) = 1023184887, \quad S(p^2) = 70112319,$$

whence

$$r_{p_1p_2} = .472 \pm .035.$$

For nitrogen,

$$S[(n-1)p] = 6458.63, \quad S[(n-1)p^2] = 13464.6031,$$
$$\bar{p} = 2.080744, \quad \sigma_p{}^2 = .008327,$$
$$S(C_p{}^2) = 14409.6095, \quad S(p^2) = 968.3721,$$

and

$$r_{p_1p_2} = .096 \pm .045.$$

Again the weighted means and standard deviations do not differ widely from those used above. The differences in the correlations will be discussed below.

In concluding this section it may be pointed out that all of the foregoing values are surprisingly high. They indicate clearly that the irregularities of an apparently uniform field may influence profoundly the yield of a series of experimental plots. They also bring out another interesting point. In the three cases in which two different characters were measured on the same species they show very different susceptibilities to environmental influence. Thus, for example, the correlation of mangold roots is $r = .346 \pm .042$ as compared with $r = .466 \pm .037$ for leaves. For grain on the Rothamsted field with a 4×5-fold grouping the correlation is $r = .186 \pm .029$ as compared with $r = .343 \pm .027$ for straw. For Montgomery's data for yield and composition the differences are

even more conspicuous. The correlation for per cent. nitrogen is $r = .115 \pm .044$ as compared with $r = .603 \pm .029$ for weight of grain produced.

This point will not be discussed in greater detail here, since the problem of the relative susceptibility of various characteristics of the individual to environmental influence has been the subject of experimental and statistical studies which have been under way for several years and will probably eventually be published.

III. On the Nature of the Regression of Associated Plots

The correlation coefficient is strictly valid as a measure of interdependence only when regression is linear, *i. e.*, when the means of the second variable associated with successive grades of the first lie in a sensibly straight line. The equation for the regression straight line

$$p_2 = \left(\bar{p}_2 - r_{p_1p_2}\frac{\sigma_{p_2}}{\sigma_{p_1}}\bar{p}_1\right) + r_{p_1p_2}\frac{\sigma_{p_2}}{\sigma_{p_1}}p_1$$

for the second on the first ultimate plot of the same combination plot reduces to

$$p = (\bar{p} - r\bar{p}) + rp,$$

when symmetrical tables in which $\bar{p}_1 = \bar{p}_2$, $\sigma_{p_1} = \sigma_{p_2}$ are used.

The testing of the linearity of regression in any individual case is rendered somewhat difficult by the necessity

TABLE V

Yield of Grain in Rothamsted Wheat Experiment

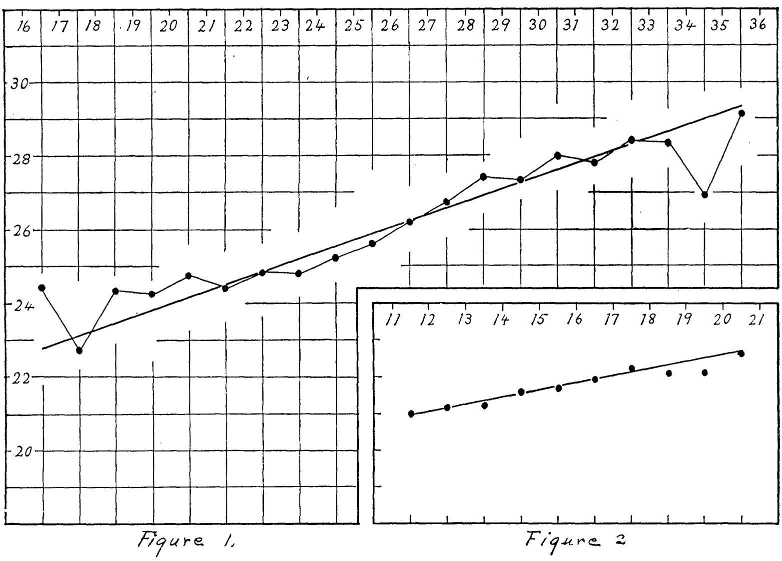
Yield of First Plot	Weighted Frequency	Mean Yield of Associated Plots	Yield of First Plot	Weighted Frequency	Mean Yield of Associated Plots
2.75–2.99	133	3.76	4.00–4.24	1786	3.99
3.00–3.24	475	3.78	4.25–4.49	1444	4.07
3.25–3.49	1026	3.81	4.50–4.74	703	4.04
3.50–3.74	1634	3.89	4.75–4.99	247	4.05
3.75–3.99	1919	3.93	5.00–5.24	133	4.16

of actually forming a correlation table from which to compute the means of arrays. The labor is greatly lessened by the use of some such scheme as that described for the formation of condensed correlation tables.[18]

TABLE VI

YIELD OF STRAW IN ROTHAMSTED WHEAT EXPERIMENT

Yield of First Plot	Weighted Frequency	Mean Yield of Associated Plots	Yield of First Plot	Weighted Frequency	Mean Yield of Associated Plots
4.00–4.24	19	6.11	6.50–6.74	608	6.56
4.25–4.49	19	5.68	6.75–6.99	817	6.69
4.50–4.74	133	6.08	7.00–7.24	779	6.86
4.75–4.99	171	6.07	7.25–7.49	665	6.84
5.00–5.24	304	6.19	7.50–7.74	627	7.04
5.25–5.49	418	6.13	7.75–7.99	323	6.96
5.50–5.74	722	6.18	8.00–8.24	247	7.14
5.75–5.99	1121	6.20	8.25–8.49	57	7.09
6.00–6.24	1273	6.31	8.50–8.74	152	6.75
6.25–6.49	969	6.38	8.75–8.99	76	7.28



FIGS. 1 AND 2. Mean Yields of Grain and Straw on Ultimate Plots Associated in the Same Combination Plots of a Given Yield. Rothamsted Wheat. Empirical Means and Fitted Straight Line. Units are Quarters of a Pound.

[18] Harris, J. Arthur, "On the Formation of Condensed Correlation Tables when the Number of Combinations is Large," AMER. NAT., 46, 477–486, 1912.

For the 5 × 4 grouping of the 500 wheat plot of Mercer and Hall I find the values given in Tables V–VI.

For the regression of the second on the first plot the equations are:

For grain, g, $\quad g_2 = 3.214 + .186\, g_1$.
For straw, s, $\quad s_2 = 4.280 + .343\, s_1$.

Figs. 1 and 2 exhibit the usual irregularities of sampling in the means, but show no certain departure from linearity.

TABLE VII

YIELD OF GRAIN IN MONTGOMERY'S WHEAT EXPERIMENT

Yield of First Plot	Weighted Frequency	Mean Yield of Associated Plots	Yield of First Plot	Weighted Frequency	Mean Yield of Associated Plots
325–374	9	516.88	575–624	111	579.82
375–424	63	440.22	625–674	90	616.21
425–474	93	471.23	675–724	45	656.37
475–524	108	540.24	725–774	30	628.80
525–574	120	548.24	775–824	3	574.00

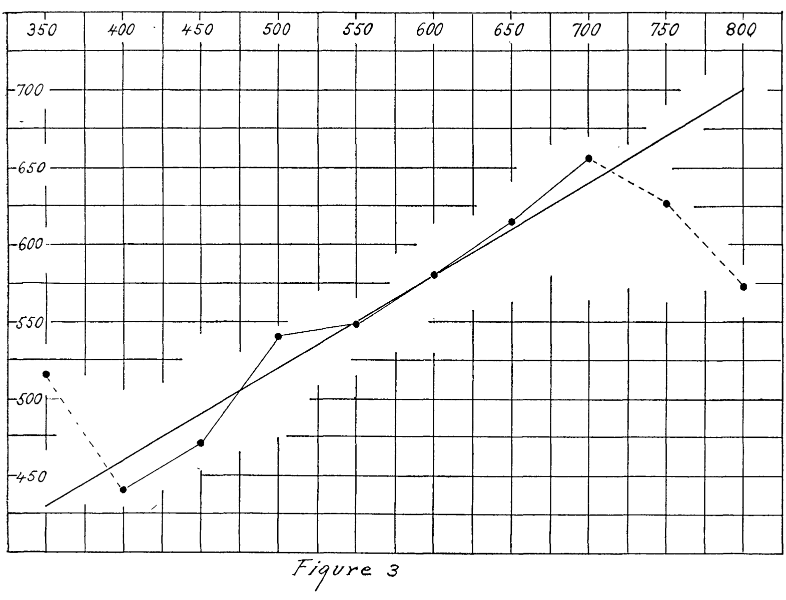


FIG. 3. Grain Yields in Nebraska Wheat. See Figs. 1–2 for Explanation.

Table VII gives the first plot character, weighted frequencies and empirical means for associated plots for 2×2-fold combinations from Montgomery's grain yield data in wheat.[19]

The equation is

For grain, g, $g_1 = 218.993 + .603\, g_1$.

The graph figures indicate sensible linearity.

IV. Influence of Number of Ultimate Plots Combined

If an experimental field exhibit irregularities of conditions which influence in a measurable degree the yield of

TABLE VIII

5×2–Fold Combination of Plots of Rothamsted Wheat

The divisions of the field are indicated by the double vertical lines and the arrows along the right margin in map B.

41.11 68.19	42.51 66.59	40.32 62.22	38.53 59.52	36.65 54.45
41.78 71.17	40.54 65.82	38.31 60.62	40.23 61.01	38.05 60.13
40.35 69.10	41.92 70.37	37.77 60.65	40.01 58.10	39.48 58.00
37.80 61.50	42.42 69.94	37.84 59.46	40.31 61.17	35.39 54.21
40.42 65.95	42.03 71.09	36.69 59.97	41.84 64.09	38.83 56.99
39.38 67.36	42.67 78.49	38.25 65.30	39.66 69.19	38.51 63.10
42.77 71.95	42.17 75.20	38.07 66.92	38.05 64.05	40.22 63.45
41.59 70.84	40.25 72.24	35.53 64.88	33.30 57.13	35.59 58.57
41.75 71.84	41.44 71.92	40.12 67.99	34.00 60.55	38.13 62.36
43.44 76.11	43.12 74.86	42.13 70.43	34.52 59.54	38.53 62.52

[19] Because of the many differences in the two experiments it is inadvisable to attempt drawing the regressions lines in a strictly comparable form.

neighboring small experimental plots, this heterogeneity should become apparently less when expressed on a scale of correlation between plots as the number of ultimate plots combined increases. The reason for this condition is quite simple. If the irregularities are very local in nature they will influence in the same direction the yield of only a very few neighboring plots. If too many ultimate plots be combined the correlation will tend to vanish because of the increased frequency of association of unlike conditions due to the fact that the combination plots have been made so large that they themselves have become heterogeneous.

That these conditions have been observed in actual experimentation is shown by the following constants based on different groupings of the data used above.

Consider first the Rothamsted wheat. For a 4×5 grouping of the plots the results were found to be

For grain, $r_{p_1p_2} = .186 \pm .029$,
For straw, $r_{p_1p_2} = .343 \pm .027$.

If the plots be grouped by fives from east to west and by twos from north to south, Table VIII is obtained. The values $S(p^2)$, $\bar{p}$ and σ_p are the same as in the preceding case.

$$m[n(n-1)] = 50 \times 10 \times 9 = 4500.$$

For grain, $S(C_p^2) = 78265.2822$, $r_{p_1p_2} = .214 \pm .029$.

For straw, $S(C_p^2) = 213939.8774$, $r_{p_1p_2} = .365 \pm .026$.

If the combination plots be made even smaller by grouping in a 2×2-fold manner for all but the last three north and south rows, where a 2×3-fold combination must be adopted, the results are, as illustrated above,

For grain, $r_{p_1p_2} = .354 \pm .026$,
For straw, $r_{p_1p_2} = .479 \pm .023$.

For Montgomery's wheat data the results for a 4×4-fold grouping (in as far as the nature of the records will permit) have been shown to be

For grain, $r_{p_1p_2} = .472 \pm .035$,
For nitrogen, $r_{p_1p_2} = .096 \pm .045$,

as compared with the following values for a 2×2-fold grouping

For grain, $r_{p_1p_2} = .603 \pm .029$,
For nitrogen, $r_{p_1p_2} = .115 \pm .044$.

Finally consider the constants deduced from the hay yields published by Holtermark and Larsen.

For a 2×2-fold grouping, $r_{p_1p_2} = .609 \pm .027$,
For a 2×4-fold grouping, $r_{p_1p_2} = .471 \pm .034$,
For a 2×8-fold grouping, $r_{p_1p_2} = .278 \pm .040$.

Thus for every species of plant and every character considered the correlation between associated ultimate plots decreases as the number of plots grouped increases.[20]

TABLES IX AND X

2×4–Fold and 2×8–Fold Combination of the Data for Plot Yield in Timothy Hay, Tables Derived from Table IV

163.8	169.2	153.5	138.9	134.9	123.0
142.0	119.3	130.6	188.6	155.4	128.6
130.9	120.3	123.3	164.2	155.0	135.9
129.8	139.1	121.5	125.0	159.1	150.8
146.3	150.0	128.6	164.6	146.7	129.9

305.8	288.5	284.1	352.8	310.4	264.5
260.7	259.4	244.8	289.6	305.8	280.7
285.2	284.9	251.6			

[20] Of course, the same effect would be produced if comparisons were drawn between tests for substratum heterogeneity on fields comparable in every regard except for the size of the ultimate plots. Possibly, this explains in part, at least, the striking differences in the correlations for grain yield found from the records of Montgomery and of Mercer and Hall.

The Rothamsted plots were 1/500th acre in area or 87.12 square feet. Montgomery's plots were $5.5 \times 5.5 = 30.25$ square feet, or only about 1/3 of the area of the Rothamsted plots.

V. Recapitulation and Discussion

If the methodical production of new varieties of animals and plants to be made possible by the laws discovered in experimental breeding is to be of material practical value, more attention must be given to the development of a standardized scientific system of variety testing. From the practical standpoint, nothing is to be gained by the formation of varieties of plants differing in discernible features of any kind unless some of these varieties can by rigorous scientific tests be shown to be of superior economic value.

It is equally true that if tests of fertilizers or of different methods of irrigation carried out on an experimental scale are to have any real value as a guide to a commercial practise, the differences in the experimental results must certainly be significant in comparison with their probable errors.

The problem of plot tests has several different phases, all of which must ultimately receive careful investigation. The purpose of this paper has been to consider one of the problems only. To what extent do the irregularities of an apparently homogeneous field selected for comparative plot tests influence the yield of the plots?

The question has been far too generally neglected, although indispensable to trustworthy results. It is obviously idle to conclude from a given experiment that variety *A* yields higher than variety *B*, or that fertilizer *X* is more effective than fertilizer *Y*, unless the differences found are greater than those which might be expected from differences in the productive capacity of the plots of soils upon which they were grown.

The first problem has been to secure some suitable mathematical criterion of substratum homogeneity (or heterogeneity). Such a criterion should be expressed on a relative scale ranging from 0 to 1, in order that com-

The 2×2-fold grouping of Montgomery's plots gives a correlation of $.603 \pm .029$ as compared with $r = .354 \pm .026$ for as nearly a perfect 2×2-fold grouping as the Rothamsted records permit.

parisons from field to field, variety to variety or character to character, may be directly made. It should also, if possible, offer no difficulties of calculation.

The criterion proposed is the coefficient of correlation between neighboring plots of the field. An exceedingly simple formula for the calculation of such coefficients has been deduced.

The method of application of this coefficient is here illustrated by four distinct series of experimental data.

The remarkable thing about the results of these tests is that in every case the coefficient of correlation has the positive sign and that in some instances it is of even more than a medium value. In short, *in every one of these experimental series the irregularities of the substratum have been sufficient to influence, and often profoundly, the experimental results.*

It might be objected that by chance, or otherwise, the illustrations are not typical of what ordinarily occurs in plot cultures. But they have been purposely drawn from the writings of those who are recognized authorities in agricultural experimentation, and who have given their assurance of the suitability of the fields upon which the tests were made.

For example, Mercer and Hall state the purpose of their research to be, "to estimate the variations in the yield of various sized plots of ordinary field crops which had been subjected to no special treatment and appealed to the eye sensibly uniform." Their mangolds "looked a uniform and fairly heavy crop for the season and soil," while in their wheat field "a very uniform area was selected, one acre of which was harvested in separate plots, each one five hundredth of an acre in area." The data of Larsen were drawn from an experiment "auf einer dem Auge sehr gleichmässig erscheinenden, 3 Jahre alten Timotheegraswiese." Montgomery's data were secured from a plot of land only 77 $\times$ 88 feet in size, which had been sown continuously to Turkey Red wheat for three

years, "and was of about average uniformity and fertility."

Nothing could, it seems to me, emphasize more emphatically the need of a scientific criterion for substratum homogeneity than the facts that correlations between the yields of adjacent plots ranging from $r = .115$ to $r = .609$ can be deduced from the data of fields which have passed the trained eyes of agricultural experimenters as satisfactorily uniform.

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